The Fundamental Importance of the Reciprocity Theorem for Observational Cosmology

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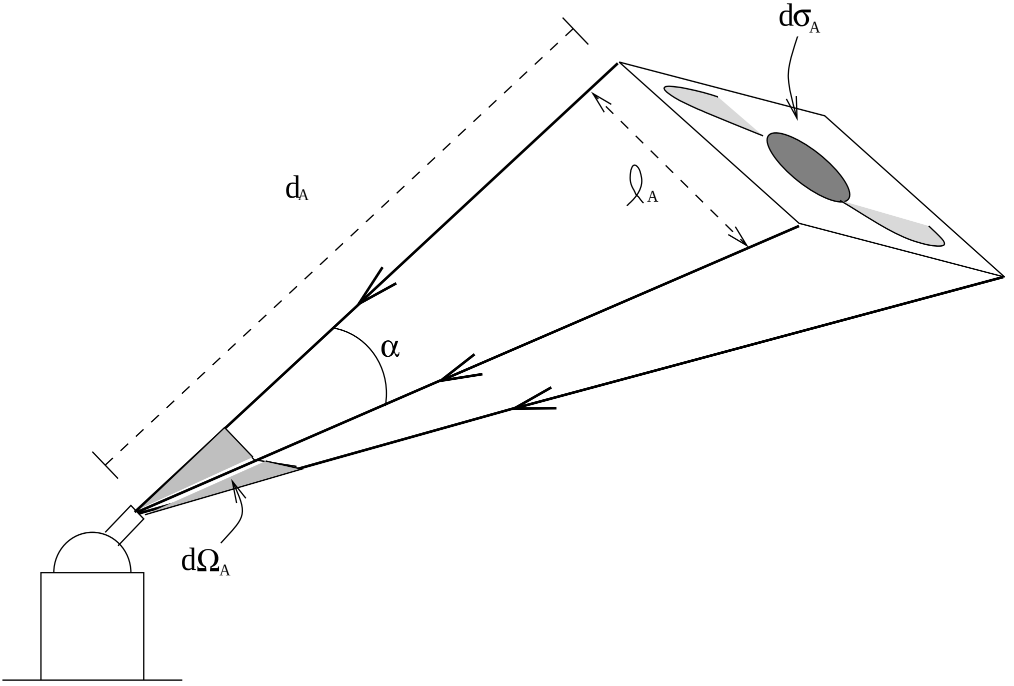
I.M.H. Etherington (1933)

- For observers at rest in a static spacetime, objects with the same size are seen at each other subtending the same solid angle;
- This is, of course, valid in Euclidean space, in this case being an intuitive result;
- In the context of a discussion with E.T. Whittaker, H.S. Ruse and R.C. Tolman about the nature of distance between objects in **general Riemannian spacetimes**, Etherington showed that for two observers **in relative motion** to one another, when redshift effects ought to be considered, the solid angle measured by these two observers to one another are changed according to the relative motion of the different frames;

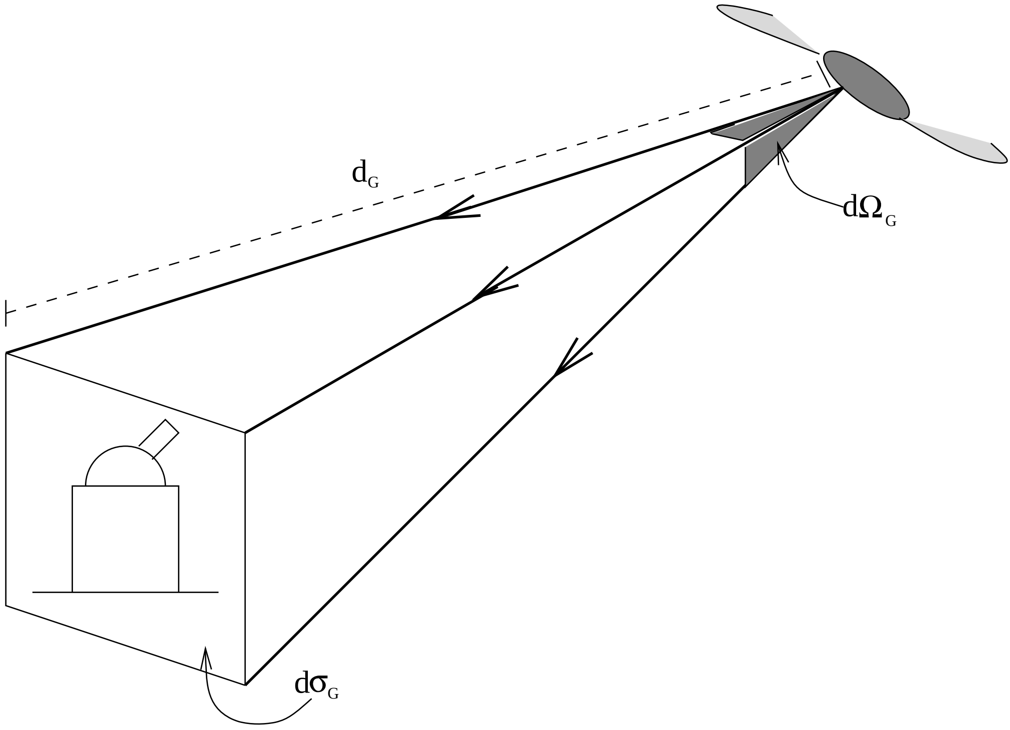
Observer's viewpoint



- Past directed (incoming) bundle of null rays subtend a solid angle $d\Omega_A$ at the observer, in the observer's time t_0, with cross sectional area $d\sigma_A$ at the observed object, a galaxy;
- This defines an area distance d_A in terms of the observer, called by various names: "*area distance*", "observer *area distance*", "*distance by apparent size*", "*angular diameter distance*" or "*corrected luminosity distance*"

Source's viewpoint

- Future directed (outgoing) bundle of null rays emanating from the source subtend a solid angle $d\Omega_G$ at the source, with cross sectional area $d\sigma_G$ at the observer at the same time t_0;
- This defines another area distance d_G, now in terms of the source, known by various names: "*galaxy area distance*", "*effective distance*", "*angular size distance*", "*transverse comoving distance*" or "*proper motion distance*"



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The general reciprocity theorem, or reciprocity law

- Etherington was the first to present a proof valid in any curved spacetime, that the way solid angles transform under velocity transformations, these two area distances are related up to redshift factors (z is measured for the source by the observer);
- ***<u>This result is valid for any spacetime, no matter how lumpy or anisotropic.</u>***

$$d\sigma_G \, d\Omega_A = (1+z)^2 \, d\sigma_A \, d\Omega_G$$

$$d_G = d_A (1+z)$$

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What is directly measurable?

- d_A can, in principle, be obtained if, as a 1st approximation, we are able to find objects whose intrinsic sizes ℓ_A are known and their angular sizes α are measured;
- However, d_G is not determinable.

$$\ell_A \approx \alpha \, d_A$$

Emission of radiation

- Considering radiation diverging from the source, d_G is, up to redshift factors, the same as the ***luminosity distance*** d_L, which is determined by measuring the flux of received radiation F_R and comparing it with the intrinsic luminosity L;
- With previous equation we can reach the ***alternative form of the reciprocity theorem***:

$$F_R = \frac{L}{4\pi\, d_L^{\ 2}}; \quad d_L = d_G(1+z)$$

$$d_L = d_A(1+z)^2$$

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Apparent luminosity (flux)

- The point like bolometric (all wavelengths) flux F_E emitted by the source and received as F_R by the observed are also related:

$$F_R = \frac{F_E}{{d_A}^2 (1+z)^4} = \frac{F_E}{d_G (1+z)_2}$$

Intensity

- Considering an extended source, or parte of a source, under area $d\sigma_A$,its emitted intensity I_E is also related by the received intensity I_R over observed solid angle $d\Omega_A$ by the reciprocity law;

$$\frac{F_R}{d\Omega_A} = \boxed{I_R = \frac{I_E}{(1+z)^4}} = \frac{1}{(1+z)^4}\frac{F_E}{d\sigma_A}$$

- Surface brightness depends only on redshift. It does not depend on any area distance.
- So, the marking density of a galaxy image in a CCD does not depend on the spacetime curvature.

Specific flux

- Considering now restricted wavelengths ranges (non bolometric quantities), the received flux F_{ν_R} is given in a certain wavelength range ν_R , $\nu_R + d\nu_R$ as

$$F_{\nu_R} d\nu_R = \frac{L}{4\pi} \frac{J[\nu_R(1+z)]}{{d_A}^2(1+z)^3} d\nu_R$$

- J is the source spectrum distribution. Emitted and received frequencies are related to the redshift by the well-known relation:

$$\nu_R(1+z) = \nu_E$$

Specific intensity and temperature shift

- Considering the specific intensity and the specific flux, we have:

$$I_{\nu_R} = \frac{I_{\nu_E}}{(1+z)^3}$$

- If a source emittes as a blackbody with temperature T_E, it is also received as a blackbody spectrum, but with temperature T_R, such that:

$$T_R = \frac{T_E}{1+z}$$

General validity of all previous results

- This set of results is easy to prove for FLRW (spatially homogeneous and isotropic) cosmology, as done by W.H. McCrea (1934, 1935) and rederived by G.C. McVittie (1956);
- But, **they are GENERAL, valid for ANY spacetime geometry**;
- This is not recognized in usual textbooks (e.g. Weinberg 1972, 2008; Peebles 1980, 1993)
- They are the result of the basic hypotheis of General Relativity theory that light travels along null geodesics in Riemannian spacetime

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Rediscovery

- Etherington's general result was lost to later generations of cosmologists, and independently rediscovered in the 1960s by R. Penrose (1966) and J. Kristian & R.K. Sachs (1966);
- G.F.R. Ellis (1971; reprinted in 2009) made these results popular in his seminal and highly influential survey on Relativistic Cosmology. A proof based on the geodesic deviation was also presented. Other proofs are mentioned in G.F.R. Ellis (2007);
- Etherington's original paper was reprinted in 2007 in the GRG Journal, in the section "Golden Oldies";
- At present **many observational cosmologists are still unware** of the generality of these results. In his recent, and highly cited, survey on distance measures, D.W. Hogg (2000) does not even mention any of these pioneering authors (although he does cite Etherington in later papers).

Fundamental result for cosmology

- The reciprocity law is key element in three areas of cosmology:

1. *Gravitational lenses*
2. *CBR observations and analysis*
3. *Galaxy observations*

Reciprocity law and gravitational lenses

- It appears in equations describing image distortion, as it uses d_A to discuss image shapes (Perlick 2004);
- It also appears indirectly when discussing brightness of images, relating surface brightness at the source $L/d\sigma_A$ and intensity at the observer I_R (Schneider, Ehlers & Falco 1992; Perlick 2004).

Reciprocity law and CBR observations

- Temperature-shift equation is fundamental to understand temperature changes in FLRW cosmologies (e.g., eq. 2.1.4 in Weinberg 2008) and is basic for discussing the evidence for the hot big-bang expansion;
- It is the foundation of Sachs & Wolf (1967) CBR anisotropy analysis (temperature-shift equation appears in eq. 40 in their paper) in perturbed FLRW cosmologies;
- Later CBR anisotropies analysis based on Liouville equation have the reciprocity law in its basis, as this equation also leads to the reciprocity theorem (Sachs & Ehlers 1971).

Reciprocity law and galaxy observations

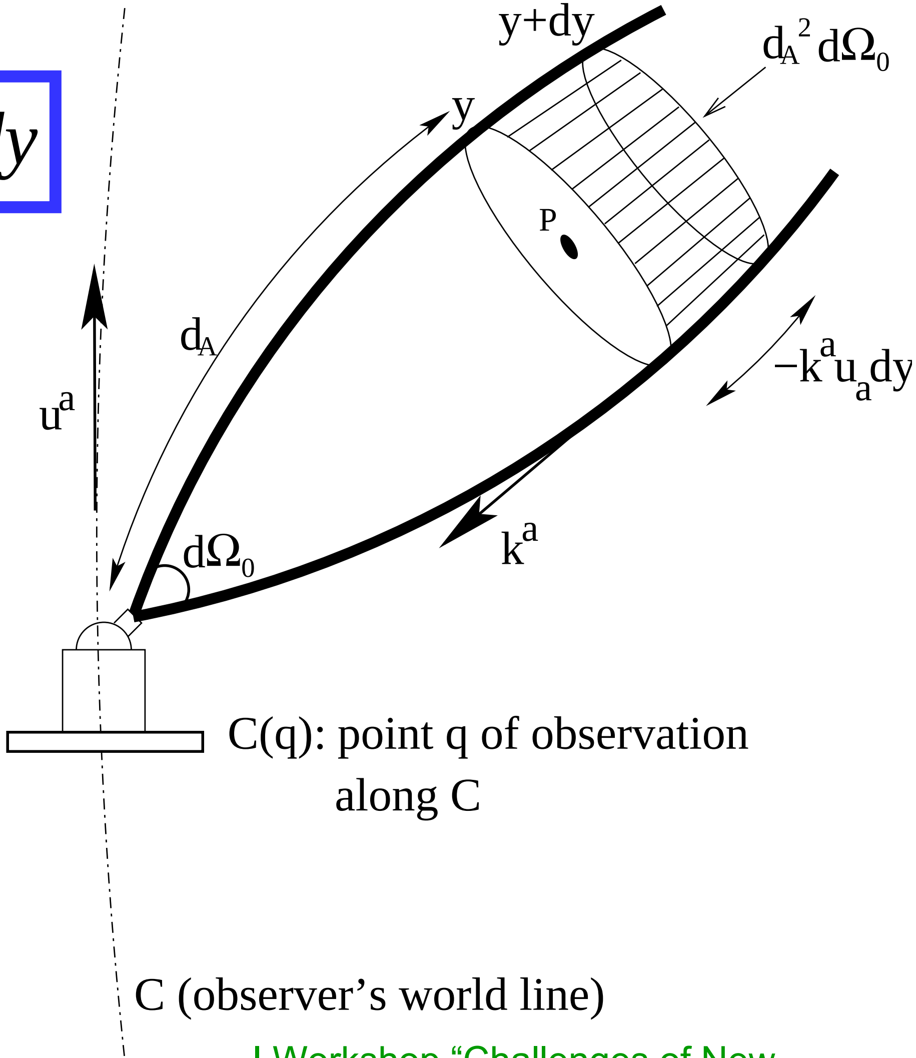
- Relation between d_L and d_A is taken for granted in galaxy observations analysis in order to eliminate one of the unknowns;
- This is valid for most of what is done in galaxy observations, from correlation analysis, number counts and luminosity function;
- otherwise it would not be possible to obtain absolute magnitudes and comoving distances from observed apparent magnitudes and redshifts;
- Galaxy detection and selection are also affected by means of number counts (Ellis 1971, 2009)

General number source counts in relativistic cosmology

$$dN = (d_A)^2 \, d\Omega_0 [n(y)(-k^a u_a)]_P \, dy$$

Ellis (1971)

- Affine parameter y along the past directed null geodesic
- Tangent vector k^a of null geodesics
- Observer 4-velocity u^a
- Source number density in proper volume n(y)
- Observer's solid angle $d\Omega_0$



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Observationally testing the reciprocity theorem

- Testable, in principle, since both d_L and d_A can be observed;
- Find two sets of homogeneous class of sources providing standard candles L (intrinsic luminosities) and standard rods ℓ_A (intrinsic sizes);
- Current best standard candle: supernova Ia data;
- Current best standard rod: BAO data;
- What we need are values with ***small*** uncertainties;
- This relation is also known in the recent literature as “Etherington distance duality”.

$$\frac{d_A}{d_L}(1+z)^2 = 1$$

Remarks

- These results do not require a cosmological model. So other geometrical distances like comoving distance, are not essential to obtain d_L and d_A;
- ***These results are based on the fundamental hypothesis of General Relativity that light travels along null geodesics in Riemannian spacetime***;
- So we only require that source and observer are connected by null geodesics.

Recent attempts to test the reciprocity law

- Despite its importance, ***the 1st attempt*** *(that I am aware of)* ***to test observationally the reciprocity law is only 5 years old !!!*** (Basset & Kunz 2004, astro-ph/0312443v2)
- The law requires photon number conservation;
- A source of "photon absorption" would dim the transparency of the Universe, since the reduced photon number would dim supernovae brightness and increase d_L, but without affecting d_A (Basset & Kunz 2004);
- The reciprocity law could also be used to test "unorthodox" gravity models, where photons do not travel along unique geodesics (Basset & Kunz 2004);
- In that case it could probe ***new physics***!

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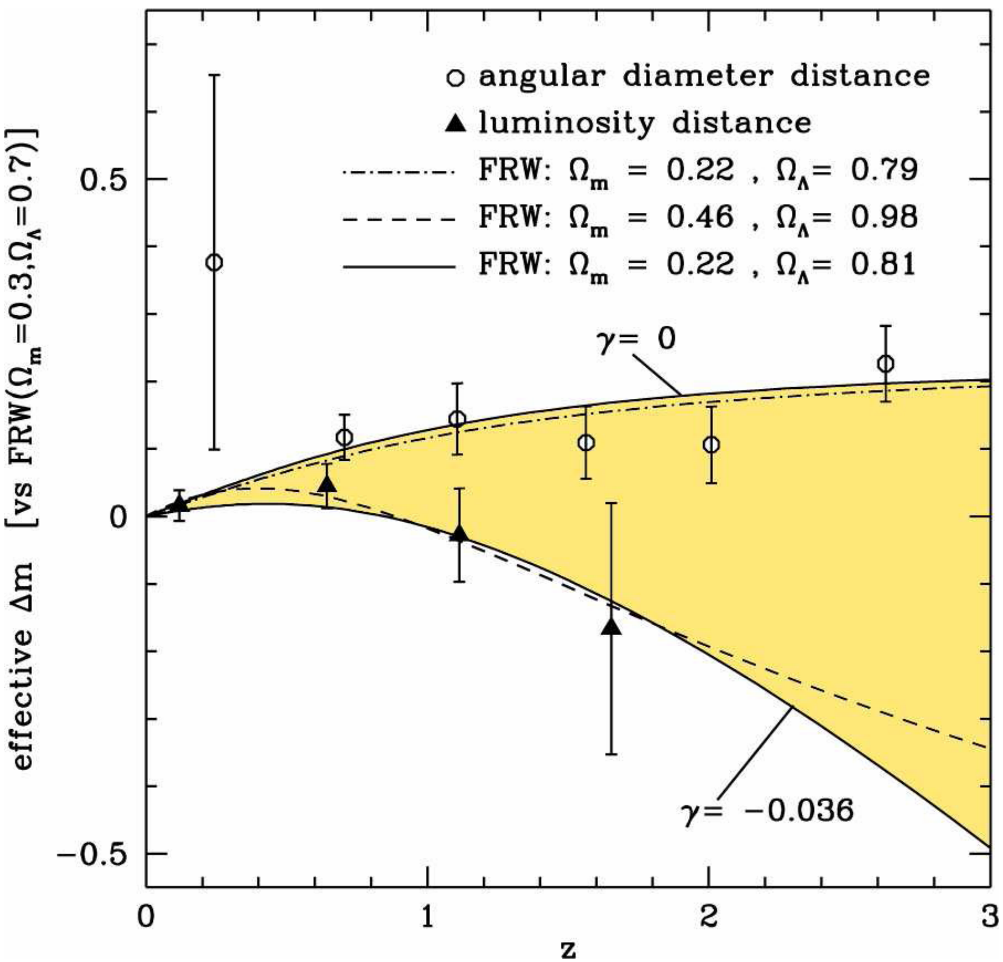


- Basset & Kunz (astro-ph/0312443v2) results;
- Inconclusive in terms of distinguishing between possible "new physics" or photon dimming;
- They assumed a Ω_Λ=0.7,Ω_m=0.3 FLRW model;
- γ=0 means no photon absorption;
- γ=-0.036 means deviations to the reciprocity law.

- Uzan, Aghanim & Mellier (2004, astro-ph/045620v1) analysed x-ray observations of clusters of galaxies and estimated d_A/d_L by means of the Sunyaev-Zel'dovich effect;
- Results in favor of the validity of the reciprocity law.

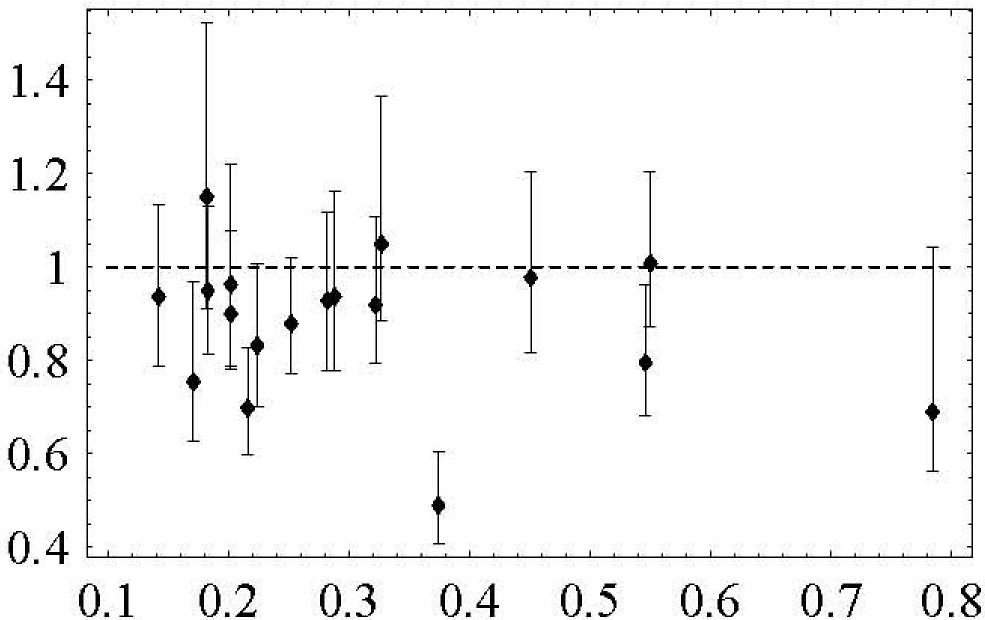
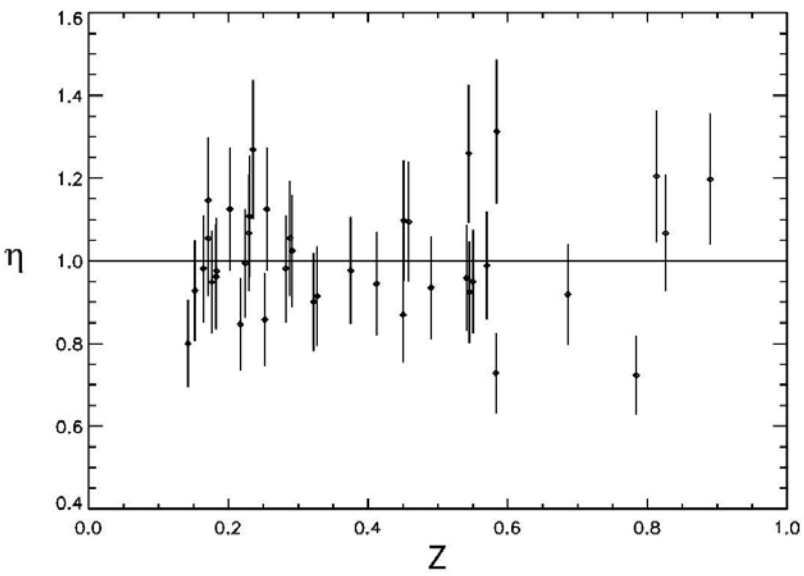


FIG. 3: η as a function of the redshift for the 18 clusters of the Reese *et al.* [27] catalog. The error bars include the observational error bars as determined by Reese *et al.* and the uncertainties in the cosmological parameters.

$$\frac{d_A}{d_L}(1+z)^2 = \eta(z)$$

- Similar test (SZ/x-ray effect in clusters combined with SN-Ia data) carried out by De Bernardis, Giusarma & Melchiorri (gr-qc/0606029v1) also presented evidence in favor of the validity of the reciprocity law



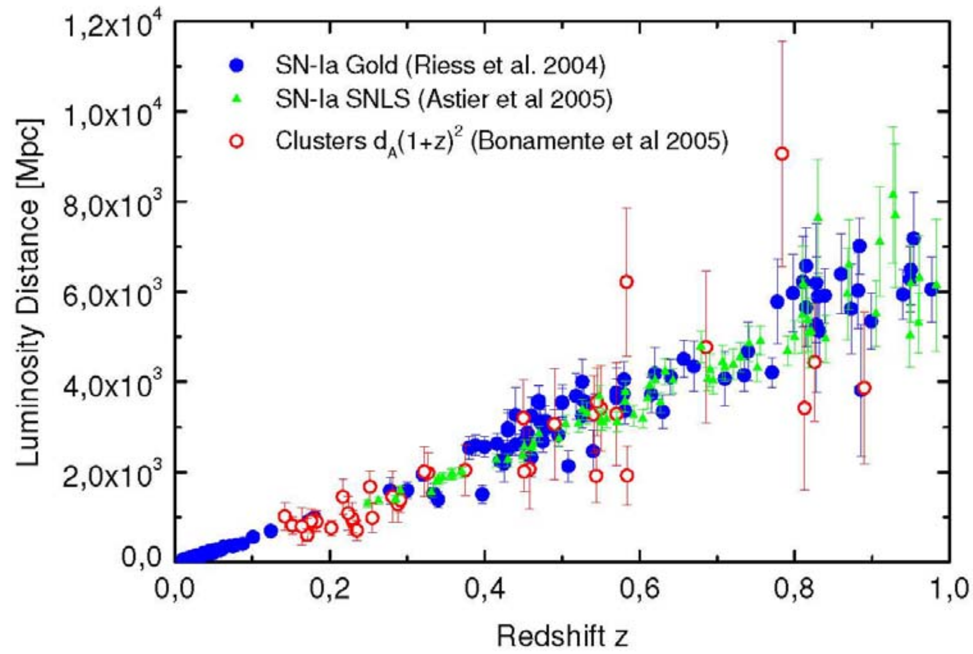


FIG. 3: Comparison of luminosity distance data from SN-Ia (Riess et al. 2004, Astier et al. 2005)) and angular distance data from SZ/X-Ray cluster observations (rescaled by $(1+z)^2$). The datasets are compatible providing no indication for systematics and/or modification of the duality distance relation.

- Results of Lazkoz, Nesseris & Perivolaropoulos (0712.1232);
- They found no statistically significant evidence for violation of the cosmic distance duality;
- They report $\eta(z)= 0.95 \pm 0.025$ in the redshift range $0 < z < 2$.

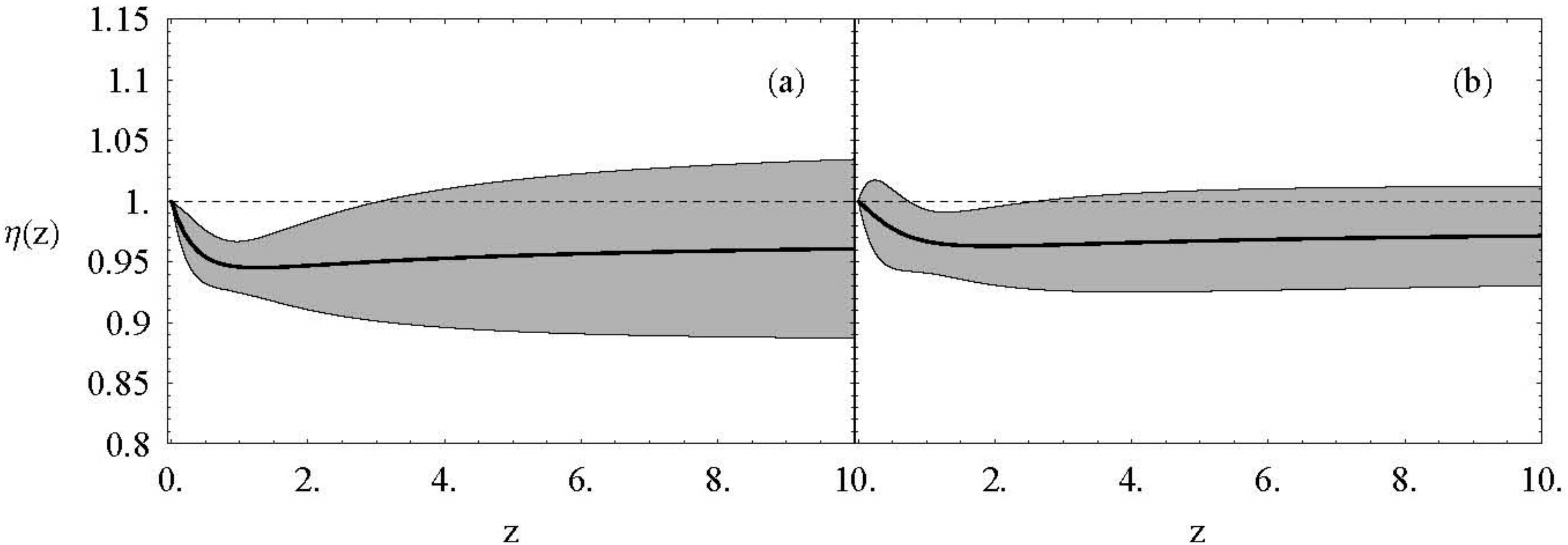


FIG. 7: The constrains on $\eta(z) \equiv \frac{d_L(z)}{d_A(z)(1+z)^2}$ for $\Omega_{0m} = 0.24$ in Fig. 7a and for $\Omega_{0m} = 0.27$ in Fig. 7b. Clearly the anticipated value $\eta = 1$ is within $1 - 2\sigma$ for both priors used.

Other recent studies attempting to test Etherington reciprocity theorem

- More, Bovy & Hogg (0810.5553) ***[inconclusive]***
- Avgoustidis, Verde & Jimenez (0902.2006) ***[in favor]***
- These are based in the possible dimming of light (transparency of the Universe);
- **However, the ideal tests should not assume any cosmological model !**

What if Etherington's reciprocity theorem does not pass the test of observations?

- Lead to a major crises in observational cosmology as distances, angle and temperature relations above are valid in ***all cosmologies***;
- The ***whole body of analysis of CMBR anisotropy would be in doubt***, as the temperature-shift relation would, presumably, no longer be valid;
- Would be ***a catastrophe to theoretical cosmology*** as we, perhaps, could no longer assume that light travels along null geodesics.